Section 4

Graphing Motion: Distance, Velocity, and Acceleration



What Do You Think?

Some automobiles can accelerate from 0 to 60 mi/h (about 100 km/h) in 5 s. Other vehicles can take up to 10 s or more to reach the same speed.

• An automobile and a bus are stopped at a traffic light. What are some differences and similarities of the motion of these two vehicles as each goes from a stop to the speed limit of 30 mi/h?

Record your ideas about this question in your *Active Physical Science* log. Be prepared to discuss your responses with your small group and the class.

Investigate

In this *Investigate*, you will use a motion detector to explore motion. You will produce distance-time and velocity-time graphs for a cart as it moves down and up an inclined ramp. You will also use the defining equation to calculate acceleration.

1. Set a motion detector at the top of a ramp along with a cart. Before collecting the data, you will make several predictions.

Florida Next Generation Sunshine State Standards: Additional Benchmarks met in Section 4

SC.912.N.1.7 Recognize the role of creativity in constructing scientific questions, methods and explanations.

SC.912.N.2.4 Explain that scientific knowledge is both durable and robust and open to change. Scientific knowledge can change because it is often examined and re-examined by new investigations and scientific argumentation. Because of these frequent examinations, scientific knowledge becomes stronger, leading to its durability.

SC.912.P.12.2 Analyze the motion of an object in terms of its position, velocity, and acceleration (with respect to a frame of reference) as functions of time.

LA.910.4.2.2 The student will record information and ideas from primary and/or secondary sources accurately and coherently, noting the validity and reliability of these sources and attributing sources of information.

MA.912.S.3.2 Collect, organize, and analyze data sets, determine the best format for the data and present visual summaries from the following: bar graphs line graphs 9 tem and leaf plots eircle graphs line tograms bo x and whisker plots • catter plots eum ulative frequency (ogive) graphs.

- ▲ a) Predict how the distance the cart travels will change with respect to time. Will it go the first half of the distance in the same amount of time as the last half of the distance?
- ▲ b) Below are four different distance-time (*d*-*t*) graphs. In one of them, the cart does not move. In the other three, the more time that elapses, the further the cart has gone. One graph shows that the cart travels at a constant speed. In another, the cart travels faster at the beginning. In another one, the cart travels fastest at the end.



Identify which graph corresponds to which motion. (Hint: Compare the distance traveled in the first few seconds of the trip with the distance traveled in the last few seconds of the trip.)

- Set C) Predict what you think a distancetime graph will look like when your cart is released from the top of the ramp. Sketch your predicted graph in your log along with an explanation.
- 2. Release the cart and collect the distancetime data. You may need to try this several times to make sure the motion detector collects consistent results.

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- a) Sketch the *d-t* graph from the calculator or computer in your log.
- b) Compare your predictions in Step 1.c) to what really happened. Explain any differences you find.
- ▲ c) In Section 3, you found that the slope of a distance-time graph represents the speed. If your graph is a curve rather than a straight line, you can still find the slope at a single point on the curve. To do this, choose the point where you want to measure the slope. Then place a ruler so that it intersects the curve at points to the right and left of the point. Slide the ruler so that it finally intersects the curve at a single point. It is now a tangent line. A tangent line is a straight line that touches a curve at only one point.





Draw the line, and you can measure the slope. The measure of that slope is equal to the speed of the cart at that point (instantaneous speed).

Look at the following lines. Which lines are tangent lines? If one of the lines is not a tangent line, sketch the curve in your log and draw the correct tangent line.

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- **d** Returning to your distance-time graph, what happens to the slope of the *d*-*t* graph as time increases? What does this tell you about the velocity?
- (As you have seen, the motion of the cart can be modeled with a distance-time graph. It could also be modeled with a velocity-time graph. A velocity-time graph shows how the velocity changes as time elapses. Predict what you think a velocity-time (v-t) graph will look like for the cart moving down the incline. Sketch it in your log along with an explanation.
- 3. Replace the cart at the top of the ramp as in Step 1. Release the cart and collect the velocity-time data. You may need to

try this several times to make sure the motion detector collects accurate data.

- (a) Sketch the *v*-*t* graph from the calculator or computer into your log. Use the "TRACE" function to label three to four data points along each line. These data points will assist you in making some calculations.
- **b**) Compare your predictions in Step 2.e) to what really happened. Explain any differences you find. Why does the graph start at 0, 0?
- (1) As time increases, what happens to the slope of the *v*-*t* graph? Why does this happen?
- \mathbf{J} d) The slope of the *v*-*t* graph is the acceleration of the cart. Acceleration is defined as the change in velocity with respect to a change in time and is expressed as follows:

Acceleration = $\frac{\text{change in velocity}}{1}$

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change in time
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This relationship can be written as an equation using symbols

$$a = \frac{\Delta \nu}{\Delta t}$$

where *a* is acceleration,

 Δv is change in velocity,

 Δt is change in time or elapsed time.

Velocity represents both speed and direction. There is an acceleration:

- if there is a change in speed over a given time,
- if there is a change in direction over a given time, or
- if there is both a change in speed and a change in direction.

Since the cart going down the ramp has no change in direction, you can think of the acceleration as a change in speed with respect to time.

What happens to the acceleration of the cart as it travels down the ramp?

- Le) Use pairs of data points from your graph to calculate the acceleration.
- 4. Prepare to run another trial. This time, move the cart to the bottom of the ramp. Practice giving the cart a push until it nearly reaches the top of the ramp. You can ignore the data for the downward motion. Before taking data, predict the following:
- A) What do you think the *d-t* graph will look like? Sketch it in your log along with an explanation.
- b) What do you think the *v-t* graph will look like? Sketch it in your log along with an explanation.
- 5. Give the cart a push and collect the data. Be sure to stop the cart on the way up if it looks like it will hit the motion detector.



- ▲ a) Sketch both the *d*-*t* and *v*-*t* graphs from the calculator or computer. Use the "TRACE" function to label three to four data points along each line.
- b) Compare your predictions in *Steps* 4.a) and b) to what really happened. Explain any differences you find.

- Sc) What happens to the slope of the *d*-*t* graph? Why does this happen?
- d) What happens to the slope of the *v*-*t* graph? Why does this happen?
- **(**) Use pairs of data points from your graph to calculate the acceleration.
- 6. Prepare to run another trial. This time, move the detector and the cart to the bottom of the ramp, pointing them both toward the top of the ramp. Practice giving the cart a push until it nearly reaches the top of the ramp. Be sure to catch the cart on the way down before it strikes the motion detector. You can ignore the data for the downward motion. Before taking data, predict the following:
- ▲ a) What do you think the *d*-*t* graph will look like? Sketch it in your log along with an explanation.
- ▲ b) What do you think the *v*-*t* graph will look like? Sketch it in your log along with an explanation.
- 7. Give the cart a push and collect the data.
- ▲ a) Sketch both the *d-t* and *v-t* graphs from the calculator or computer. Use the "TRACE" function to label three to four data points along each line.
- b) Compare your predictions in *Steps* 6.a) and b) to what really happened.
 Explain any differences you find.
- Sc) What happens to the slope of the *d-t* graph? Why does this happen?
- d) What happens to the slope of the *v*-t graph? Why does this happen?
- Le) Use pairs of data points from your graph to calculate the acceleration.
- 8. On the next page, you are provided with four graphs. Describe a motion of a cart on an incline that could produce each of these graphs. Include where the motion detector would have to be placed to produce the graph.

Driving the Roads



- 9. You will now take a closer look at acceleration in a straight line. Look at the automobile data provided at the end of this chapter after *Section 7*. The tables contain a lot of information including fuel economy, passenger accommodations, acceleration, and braking. In this section, you will be concerned with acceleration.
- ▲ a) Record in your log where the acceleration information is located on the automobile table.
- 10. The speed on the table provided by automobile manufacturers is given in miles per hour (mi/h or mph), but the distances are recorded in feet and the time in seconds. To analyze this data more easily, it is helpful to record the speed in feet per second (ft/s). The table at right converts miles per hour to feet per second. Note that there are 60 min in 1 h and 60 s in 1 min. You should also note that there are 5280 ft in 1 mi. When you convert 60 mi/h to 88 ft/s, the conversion looks like the following:

$$\left(60\frac{\mathrm{mi}}{\mathrm{h}}\right)\left(\frac{1 \mathrm{h}}{60 \mathrm{min}}\right)\left(\frac{1 \mathrm{min}}{60 \mathrm{s}}\right)\left(\frac{5280 \mathrm{ft}}{1 \mathrm{mi}}\right) = 88\frac{\mathrm{ft}}{\mathrm{s}}$$

If you deal with the units in the same way that you deal with the numbers, you will see that the miles cancel miles, hours cancel hours, and minutes cancel minutes.

$$\left(60\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{1\,\text{h}}{60\,\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\left(\frac{5280\,\text{ft}}{1\,\text{mi}}\right) = 88\,\frac{\text{ft}}{\text{s}}$$

You should notice that to convert 60 mi/h to 88 ft/s, the 60 mi/h was multiplied by fractions that always equaled 1 (for example, 1 h and 60 min are the same value of time). Multiplying by $\frac{1}{1}$ keeps the value the same.

The following table was constructed on a spreadsheet. You can use the conversions in this table to give you a sense of the different units and to help you answer some of the questions in this chapter.

	А	В	С	D		
1	Common Speed Conversions					
2	United States		Canada			
3	mi/h	ft/s	m/s	km/h		
4	0	0	0	0		
5	10	15	5	16		
6	20	29	9	32		
7	30	44	13	49		
8	40	59	18	65		
9	50	73	23	81		
10	60	88	27	97		
11	70	103	31	113		
12	80	117	36	130		
13	90	132	41	146		
14	100	147	45	162		

11. The sports car's acceleration data from the table at the end of the chapter is shown below with miles per hour changed to feet per second.

Acceleration Data of a Sports Car in Feet per Second				
Final speed (ft/s)	Total time (s)			
0	0.0			
44	2.0			
59	2.9			
73	4.2			
88	5.2			
103	6.6			
117	8.7			
132	10.9			
147	13.3			

- ▲ a) Sketch a graph of speed vs. total time and label it "Velocity-Time Graph." Put the time on the *x*-axis (horizontal) and the speed on the *y*-axis (vertical). Plot your points from the table using feet per second (ft/s) units for velocity.
- b) During which time interval is the velocity changing the most?
- Sc) During which time interval is the velocity changing the least?
- Acceleration is defined as the change in velocity for each time interval. Where is acceleration the greatest? Where is acceleration the least?
- 12. You can now calculate the acceleration for each time interval.

The acceleration is equal to the change in velocity (final speed – initial speed) divided by the change in time.

$$a = \frac{\Delta v}{\Delta t}$$
$$= \frac{v_{\rm f} - v_{\rm i}}{\Delta t}$$

Where *a* is acceleration,

 Δv is change in velocity,

 $v_{\rm f}$ is final velocity,

- v_{i} is initial velocity,
- Δt is change in time or elapsed time.

The first acceleration calculation is shown below.

$$a = \frac{\Delta v}{\Delta t}$$
$$= \frac{v_{\rm f} - v_{\rm i}}{\Delta t}$$
$$= \frac{44 \text{ ft/s} - 0 \text{ ft/s}}{2 \text{ s}}$$
$$= \frac{22 \text{ ft/s}}{\text{s}}$$

The acceleration is equal to 22 feet per second every second. This is a change in speed (22 ft/s) with respect to time (1 s). This can also be written in the following ways:

- 22 ft/s every s
- 22 ft/s per s
- 22 (ft/s) per s
- 22 ft/s² (feet per second squared)

The last way is the easiest to say, but the first way is the easiest to understand.

If the automobile moved at a constant acceleration of 22 ft/s every second, you would see a constant increase in the speed every second, from 0 ft/s to 22 ft/s, then to 44 ft/s, and then to 66 ft/s. A constant acceleration is what happened to the cart on the ramp. However, this increase is not what usually happens to an automobile. An automobile does not move at a constant acceleration.

▲ a) You can calculate the acceleration for the next time interval by calculating the acceleration of the sports car from 44 ft/s to 59 ft/s. This change in speed required 0.9 s. Complete this calculation. Did you get the value in the table of 16 ft/s every second?

S	Calculating A	Acceleration of a	ed
Initial Speed (ft/s)	Final Speed (ft/s)	Change in time (5)	Acceleration (ft/s every second)
0	44	2.0	22
44	59	0.9	16
59	73	1.3	
73	88	1.0	
88	103		
103	117		
117	132		
132	147		

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- b) Work with your group members to complete the Calculating Acceleration of a Sports Car in Feet per Second Squared table in your log.
- **(**) Compare the table with the velocitytime graph you sketched in *Step 11.a*).

Recall that the slope of the velocity-time graph is equal to the acceleration. Where does the table indicate the greatest acceleration took place? Where does the graph have the steepest slope?

Physics Talk

CHANGING SPEED

Acceleration

When things change speed, it is usually noticeable. The motion of a falling object is a common example of something changing speed. Galileo Galilei, an Italian scientist, was the first person to apply mathematics to the study of the change in the speed of a falling object. To describe the change quantitatively, he needed to make measurements. In the *Investigate*, you used a motion detector to make measurements of **acceleration**. Since Galileo lived in the 1600s, he did not even have access to clocks and had to devise original ways to measure time with accuracy and precision. One technique he used involved a water clock. In a water clock, water flows through a funnel into a bowl. The more time that elapses, the more water is collected.

To help him explore falling objects, he first investigated balls rolling down an incline. He thought that a ball rolling down an incline was like watching a falling object in "slow motion."

Through his experimentation with balls rolling down inclines, Galileo found that if he looked at the change in speed with respect to the change in time, the value remained the same as the ball descended the ramp. He then defined this as acceleration. The definition of acceleration as change in velocity with respect to time is still in use today.



In this section, you observed, just as Galileo did with rolling balls, that a cart traveling down an inclined plane has a constant acceleration. The velocity of the cart changes at a regular rate and is represented by a straight line on the velocity vs. time graph.

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Physics Words

acceleration: the change in velocity with respect to a change in time.

Acceleration Is a Vector Quantity

Acceleration means "how fast the velocity changes." You will recall that the word velocity means "how fast an object is going (speed) and in what direction." Velocity, therefore, is a **vector** quantity. A vector quantity is a quantity that has both magnitude (size) and direction. A bus and an automobile can each accelerate by changing speed from 0 to 60 mi/h (about 100 km/h) and from 60 to 0 mi/h when braking, and both can change velocity by driving around curves. But the automobile can produce these velocity changes in much less time. The automobile can exhibit greater acceleration than the bus.

The distinction between speed and velocity becomes important when changes in direction can occur. For example, when driving on curves, you can have changes in the direction, and thus a change in velocity, even while maintaining a steady speed. For example, a person driving around a curve at a steady speed of 15 m/s is accelerating. There is no change in speed, but there is a change in direction.

So the ways to change your automobile's velocity are

- to speed up (increasing the speed, or magnitude of velocity),
- to slow down (decreasing the speed, or magnitude of velocity), or
- turn (change the direction of velocity).

And, of course, you can change speed and direction simultaneously, as when you drive on mountain roads with curves.

All of these motions involve accelerations, because the velocity changes as time elapses. In this section, acceleration for an automobile moving along a straight line (no curves or turns) is discussed. You will investigate changing directions later in this chapter. For now, consider motion in a straight line.

In one part of the *Investigate*, you observed a cart going up a ramp. In this case, the final velocity (at the top of the incline) was less than the initial velocity. This is a **negative acceleration**. You may have heard the word deceleration used to describe something that is slowing down. However, in physics, that term is not used. The vocabulary used to describe a change in velocity with respect to time is **positive acceleration** and negative acceleration. The precision of these terms avoids confusion that may arise when the common word, deceleration, is used.

Physics Words

vector: a quantity that has both magnitude and direction.

negative acceleration: a decrease in velocity with respect to time. The object can slow down (20 m/s to 10 m/s) or speed up (-20 m/s to -30 m/s).

positive acceleration: an increase in velocity with respect to time. The object can speed up (20 m/s to 30 m/s) or slow down (-20 m/s to -10 m/s).





You will investigate negative acceleration further in *Section 5*. For motion in a straight line, positive acceleration means that the velocity of the object is increasing over time. Negative acceleration means that the velocity of the object is decreasing over time, if the object is moving in a straight line.

Vector and Scalar Quantities

A quantity that involves both direction and size (magnitude) is called a vector quantity. A quantity that has size, but not direction, is called a scalar quantity. Speed is a scalar quantity. It only indicates the change in position over a period of time in a straight line. Velocity is a vector quantity. It can indicate a change in position over a period of time and the direction.

Describing Accelerated Motion Using Strobe Pictures

Recall that you used three different models to describe motion: strobe pictures, graphs, and equations. Each gives the same information, but in different forms. You will use the same models to describe acceleration.

Because the speed is always changing during constant acceleration, the strobe illustration below shows the automobiles moving greater distances during each second of travel.







Describing Acceleration Using an Equation

In the *Investigate*, you used an equation to describe acceleration. You calculated acceleration by finding the change in velocity with respect to time.

Acceleration = $\frac{\text{change in velocity}}{\text{change in time}}$

This relationship can be written as an equation using symbols.

$$a = \frac{\Delta v}{\Delta t}$$

where *a* is acceleration,

 Δv is change in velocity,

 Δt is change in time or elapsed time.

Units for Measuring Acceleration

To calculate acceleration, you divide change in velocity by change in time $\frac{\Delta v}{\Delta t}$. The units for acceleration are then, by definition,

velocity divided by time. Recall from the previous section, the units for velocity can be m/s or km/h. Assume that the time interval is measured in seconds. The units for acceleration would then be (m/s)/s or (km/h)/s. The change in velocity is given in meters per second every second, or kilometers per hour every second.

When writing the units for acceleration, the final units are often simplified. For example, the following all mean the same thing. The simplified units are read as meters per second squared.

$$\frac{m/s}{s}, \text{ or } (m/s)/s = \frac{m}{s^2} \text{ or } m/s^2$$

In the *Investigate*, you calculated acceleration in feet per second every second, or feet per second squared (ft/s²).

Using the Equation for Acceleration to Find Other Quantities

The defining equation for acceleration shows the relationship between acceleration, velocity, and time. If you know two of these, you can find the third.

Acceleration = $\frac{\text{change in velocity}}{\text{change in time}}$

Using algebra, it follows that

Change in velocity = acceleration \times time Time = <u>change in velocity</u>

acceleration

Using symbols, these equations can be written as

$$a = \frac{\Delta v}{\Delta t}$$
$$\Delta v = a \times \Delta t$$
$$\Delta t = \frac{\Delta v}{2}$$



Active Physical Science



As you did with the equations for speed in the previous section, you may find it helpful to use a circle, like the following:

$\begin{array}{c|c} \Delta v \\ a & \Delta t \end{array}$

By covering up the variable you wish to find, you can see the equation. To find change in velocity (Δv), cover up the Δv , and you see $a \times \Delta t$.

To find acceleration (a), cover up the a, and you see $\frac{\Delta v}{\Delta t}$.

To find time (Δt), cover up the Δt and you see $\frac{\Delta v}{a}$.

There is only one definition of acceleration. Algebra allows you to write it in different forms.

Sample Problem

At the start of a race, a toy car increases speed from 0 m/s to 5.0 m/s as the clock runs from 0 s to 2.0 s. Find the acceleration of the toy car.

Strategy: Use the definition of acceleration as the change in velocity over a change in time.

Final velocity $(v_f) = 5.0 \text{ m/s}$

Final time $(t_f) = 2.0 \text{ s}$

Initial time $(t_i) = 0$ s

Initial velocity $(v_i) = 0$ m/s



Given:

Solution: $a = \frac{\Delta v}{\Delta t}$ $= \frac{v_f - v_i}{t_f - t_i}$

 $= \frac{v_{f} - v_{i}}{t_{f} - t_{i}}$ $= \frac{5.0 \text{ m/s} - 0 \text{ m/s}}{2.0 \text{ s} - 0 \text{ s}}$ $= \frac{5.0 \text{ m/s}}{2.0 \text{ s}}$ $= 2.5 \text{ m/s}^{2}$

The acceleration is 2.5 m/s every second, and can be written and stated in three equivalent ways:

- 2.5 meters per second every second, or
- 2.5 (m/s)/s (meters per second per second), or
- 2.5 m/s² (meters per second squared).

Physics Words

tangent line: a straight line that

Describing Acceleration Using Graphs

A third way to represent acceleration is with graphs. If distance is represented on the *y*-axis and the time is represented on the *x*-axis, then the graph showing constant acceleration is a curve. The slope of the **tangent line** to the curve at any point gives the instantaneous speed at that point. One such tangent is shown on the graph below. If you imagine tangents at different points, you can see that the slopes of the tangents increase as time increases. Thus, the speed is increasing during this time. An increasing speed during a time interval is an acceleration.

If the velocity is represented on the *y*-axis and the time is represented on the *x*-axis, then the slope of the graph will be equal to the change in velocity with respect to time. The acceleration is equal to the value of the slope of the velocity-time graph. Notice that the slope has the same value at all points. You can conclude that since the slope of the *v*-*t* graph is constant, the acceleration is constant.

Describing Types of Motion Using Graphs

You can determine the general motion of an automobile by reviewing the distance vs. time graph, the corresponding velocity vs. time graph, and the corresponding acceleration vs. time graph. Each column in the table on the following page provides a way to describe the motion of an automobile.

Note: When interpreting graphs, you must always check to see if the *y*-axis represents distance, velocity, or acceleration. A horizontal line on a graph has very different meanings if the graph is a *d*-*t* graph, a *v*-*t* graph, or an *a*-*t* graph.

Use the Comparing Motion Graphs table on the next page to determine the general motion of an automobile. All of the three graphs in a column represent the same motion of the same car. One column gives the information in terms of changes in distance, another represents velocity, and the third acceleration over time.





Checking	Up

- 1. Give the defining equation for acceleration in words, and by using symbols.
- 2. What is an SI unit for measuring acceleration? Use words and unit symbols to describe the unit.
- 3. What is the difference between a vector and a scalar quantity?
- 4. Sketch a distancetime graph for
 - a) constant velocity
 - b) constant acceleration
- 5. What does the slope of a velocitytime graph represent?

	Comparing Motion Graphs					
	Automobile at rest	Automobile with constant velocity	Automobile with constant acceleration			
<i>d-t</i> graph	Distance (position) does not change.	Distance (position) changes at a constant rate.	Distance (position) changes at a non- constant rate (the <i>d-t</i> graph is a curve, not a straight line).			
	distance (d) time (t)	distance (d) time (t)	distance (d) time (t)			
<i>v-t</i> graph	Velocity is always 0 because the position does not change.	The change in distance vs. time is constant and therefore the velocity is constant rate.	The speed is increasing as can be seen by the increasing slope of the distance vs. time graph.			
	celocity (t)	time (t)	velocity (v) time (t)			
<i>a-t</i> graph	Acceleration is always 0 because velocity does not change.	There is a constant velocity. With no change in velocity, there is 0 acceleration.	The velocity is changing at a constant rate. The automobile is moving at a constant acceleration.			
	acceleration (<i>a</i>) time (<i>t</i>)	acceleration (<i>a</i>) time (<i>t</i>)	acceleration (a)			

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				Active Physics
+Math	+Depth	+Concepts	+Exploration	
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Determining Distance Using the Acceleration Equation

The definition of acceleration provides the relationship between velocity, acceleration, and time. If you know the acceleration, you can determine the change in velocity after a given time has elapsed by using the following equation:

$$\Delta v = a \Delta t$$

If an automobile has a constant acceleration, you can also determine the distance traveled after a given time has elapsed.

Knowing the initial and final velocity, you can now determine the average velocity $(\overline{\nu})$. For a constant acceleration, the average velocity is determined the same way the average of any two numbers is determined.

$$\overline{v} = \frac{v_{\rm f} + v_{\rm i}}{2}$$

Once you know the average velocity, you can use the definition of average velocity to determine the distance traveled.

Using some algebra, you can also determine the distance traveled in one step with a newly derived equation.

$$d=\overline{v}t$$

You can now determine how an object's position and velocity depend on the elapsed time from the definition of velocity and acceleration.

$$d = \overline{v}t$$
$$d = \left(\frac{v_{\rm f} + v_{\rm i}}{2}\right)t$$

Since
$$v_{f} - v_{i} = at$$

then $v_{f} = v_{i} + at$
$$d = \left(\frac{(v_{i} + at) + v_{i}}{2}\right)t$$
$$d = \left(\frac{(at) + 2v_{i}}{2}\right)t$$
$$d = \frac{1}{2}at^{2} + v_{i}t$$

Sample Problem

An automobile accelerates from rest at 5.0 m/s every second (5.0 m/s²). How far does it travel after 3.0 s?

Given:

Initial velocity $(v_i) = 0$ m/s Acceleration (a) = 5.0 m/s² Time $(\Delta t) = 3.0$ s

Strategy 1:

Find the final velocity using the definition of acceleration; then find the average velocity; and then use the relationship between distance, average velocity, and time.

Solution:

$$v_{\rm f} = at + v_{\rm i}$$
$$v_{\rm f} = \left(5.0 \ \frac{\rm m}{\rm s^2}\right) (3.0 \ \text{s}) + 0$$
$$v_{\rm f} = 15 \ \text{m/s}$$



Driving the Roads

Knowing that the final velocity is 15 m/s and the initial velocity equals 0, you can calculate the average velocity.

$$\overline{v} = \frac{v_f + v_i}{2}$$
$$= \frac{15 \text{ m/s} + 0 \text{ m/s}}{2}$$
$$= 7.5 \text{ m/s}$$

Using the definition of average velocity, the distance can be computed:

$$d = \overline{v}t$$
$$= 7.5 \frac{m}{s} \times 3.0 s$$
$$= 22.5 m$$

Strategy 2:

Because acceleration, time, and initial velocity are provided, use the derived relationship of distance, acceleration, and time. There is no need to find the final velocity.

Solution:

$$d = \frac{1}{2}at^{2} + v_{i}t$$
$$= \frac{1}{2}\left(5\frac{m}{s^{2}}\right)(3s)(3s) + 0$$
$$= 22.5 m$$



What Do You Think Now?

At the beginning of this section, you were asked the following:

• An automobile and a bus are stopped at a traffic light. What are some differences and similarities of the motion of these two vehicles as each goes from a stop to the speed limit of 30 mi/h?

How would you answer this question now? Now that you have investigated change in velocity over time, compare and contrast the motion of the vehicles using the term acceleration. Sketch a velocity-time graph for each vehicle.

Active Physical Science



What does it mean?

One race car has a greater acceleration than a second race car. But the second race car can reach a higher top speed than the first. How is this possible?

How do you know?

As you enter the highway, your automobile goes from rest to the speed limit. What measurements would you have to take to calculate the acceleration of your automobile as it enters the highway?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	* Models	Experimental evidence is consistent with models and theories

* Physicists use models to better understand the world. How can a distance vs. time graph, a velocity vs. time graph and an acceleration vs. time graph all represent a car moving with a constant acceleration?

Why should you care?

Safe driving saves lives. How can your understanding of velocity and acceleration help you to become a safer driver? In the unfortunate possibility that you are in an accident, how could d-t, v-t, and a-t motion graphs help you explain why the accident was not your fault?

Reflecting on the Section and the Challenge

Driving is all about accelerations. Automobiles accelerate when they speed up, slow down, or make a turn. Drivers depend on negative accelerations to avoid accidents when they apply the brakes. Speeding up too quickly can also lead to accidents.

You now know how to calculate accelerations and to determine accelerations from a velocity-time graph. You may want to use these calculations and/or graphs in your description of safe driving, or perhaps in your presentation.





Physics to Go

- 1. Can a situation exist in which an object has zero acceleration and nonzero velocity? Explain your answer.
- 2. Can a situation exist in which an object has zero velocity and nonzero acceleration, even for an instant? Explain your answer.
- 3. If two automobiles have the same acceleration, do they have the same velocity? Why or why not?
- 4. If two automobiles have the same velocity, do they have the same acceleration? Why or why not?
- 5. Can an accelerating automobile be overtaken by an automobile moving with constant velocity?
- 6. Is it correct to refer to speed-limit signs instead of velocity-limit signs? Why or why not? What units are assumed for speed-limit signs in the United States?
- 7. Suppose an automobile were accelerating at 2 mi/h every 5 s and could keep accelerating for 2 min at that rate.
 - a) How fast would it be going at t = 2 min?
 - b) How far would it be from the starting line?
- 8. At an international auto race, a race car leaves the pit after a refueling stop and accelerates uniformly to a speed of 75 m/s in 9 s to rejoin the race.
 - a) What is the race car's acceleration during this time?
 - b) What was the race car's average speed during the acceleration?
 - c) How far does the race car go during the time it is accelerating?
 - d) A second race car leaves after its pit stop and accelerates to 75 m/s in 8 s. Compared to the first race car, what is this race car's acceleration, average speed during the acceleration, and distance traveled?
- 9. During a softball game, a player running from second base to third base reaches a speed of 4.5 m/s before she starts to slide into third base. When she reaches third base 1.3 s after beginning her slide, her speed is reduced to 0.6 m/s.
 - a) What is the player's acceleration during the slide?
 - b) What was the distance of her slide?
 - c) If she had slid for only 1.1 s, how fast would she have been moving when she reached third base? (Assume she had the same acceleration as before.)
 - d) Which of these two trials would get her from second base to third base faster?

- 10. Suppose an astronaut on an airless planet is trying to determine the acceleration of an object that is falling toward the ground. She has a motion detector in place that records the graph to the right for the falling object until just before it strikes the ground.
 - a) From the graph, approximately what was the top speed recorded by the astronaut for the falling object?



- b) What is the acceleration of gravity on this planet?
- c) If the astronaut had dropped the object from a greater height, what would happen to the object's acceleration as it falls and the object's final velocity before striking the ground?
- 11. A boy riding a bike with a speed of 5 m/s across level ground comes to a small hill with a constant slope and lets the bike coast up the hill. All graphs have time on the *x*-axis.



- a) Which of the graphs would correctly show the boy's velocity versus time as he coasts up the hill?
- b) Which graph shows the distance traveled versus time as he coasts up the hill?
- c) Which graph would show the bike's acceleration as it coasts uphill?
- d) Which graph shows after reaching the top of the hill, the speed of the boy as he coasts down the hill on the bike?
- e) Which graph could show the boy's speed versus time graph as the boy coasts up the hill and then down the hill?
- f) Starting from the top of the hill, which graph could correctly show the boy's distance vs. time as he goes down the hill?



12. An automobile magazine runs a performance test on a new model car, and records the graph of distance versus time as the car goes around a track. During which segment or segments of the graph is the car



- a) traveling with constant speed?
- b) increasing speed?
- c) at rest?
- d) decreasing speed?
- e) How far did the car travel during the total test?
- f) According to the graph, where was the car when the test was completed?
- 13. A jet taking off from an aircraft carrier goes from 0 to 250 mi/h in 30 s.
 - a) What is the jet's acceleration?
 - b) If after take-off, the jet continues to accelerate at the same rate for another 15 s, how fast will it be going at that time?
 - c) How much time does it take for the jet to reach 500 mi/h?
 - d) How much distance would it take for that same jet to reach 500 mi/h?
- 14. Whenever air resistance can be neglected or eliminated, an object in freefall near Earth's surface accelerates vertically downward at 9.8 m/s² due to Earth's gravity. This acceleration is also called 1 g.
 - a) If the object falls for 100 m, how fast is it traveling?
 - b) How much time is required for it to fall this 100 m?
 - c) If the object falls for 10 s, how fast is it traveling?
 - d) How far has it fallen in this 10 s?
 - e) How would your answers to these questions change for an object falling above the Moon, where the acceleration is about $\frac{1}{g}$ (1.6 m/s²)?
- 15. In 1954, in a study of human endurance prior to the manned space program, Colonel John Paul Stapp rode a rocket-powered sled that was boosted to a speed of 632 mi/h (1017 km/h). The sled and he were then decelerated to a stop in 1.4 s.
 - a) What was the acceleration of this stop?
 - b) What is this acceleration in terms of g's?
 - c) In what distance did the speed of the sled travel as its speed changed from 1017 km/h to 0?

- 16. Active Physics An automobile accelerates from rest at 4.0 m/s every second (4.0 m/s^2) .
 - a) How far does it travel after 1.0 s?
 - b) How far does it travel after 2.0 s?
 - c) How far does it travel after 3.0 s?
 - d) How far does it travel after 4.0 s?
 - e) Complete a *d*-*t* graph for this automobile.
 - f) Complete a *v*-*t* graph for this automobile.
 - g) How does the motion of this automobile compare with the motion of a real automobile (as you investigated previously)?

17. Preparing for the Chapter Challenge

On highways, you can pass slower-moving vehicles by moving into the left lane and driving past them. You can then return to the right lane, all the while traveling at the speed limit. On a rural road, you must do this by entering the oncoming traffic lane. This can be very dangerous. To pass the vehicle safely and quickly, you may have to accelerate until you get back into your lane. You can describe the motion of both vehicles by creating graphs with two lines on each one—one depicting your vehicle and the other depicting the slower-moving vehicle. Describe how you can safely pass a slower-moving vehicle using *d-t*, *v-t*, and *a-t* graphs to convince the driving academy that you understand safe driving.

Inquiring Further

Speed conversions

Using a spreadsheet program, complete a table like the one below that converts mph to ft/s to m/s to km/h.

	А	В	С	D
1	mi/h	ft/s	m/s	km/h
2	0			
3	5			
4	10			
5	15			
6	20			
7	25			
8	30			
9	35			
10	40			
11	45			
12	50			